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STOCHASTIC DISCOUNT FACTOR FOR MEXICO AND CHILE, 2007-2012

Área de investigación: Finanzas

Humberto Valencia Herrera
Instituto Tecnológico y de Estudios Superiores de Monterrey
México
humberto.valencia@itesm.mx, hvalencia00@yahoo.com.mx

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Abstract

The stochastic discount factor, used in multiple applications in Finance, changed its properties during the credit crisis period in México and Chile. The paper shows how linear estimates change using the method of moments under different assumptions during the period of study.

El factor de descuento estocástico, usado en múltiples aplicaciones en Finanzas, cambio sus propiedades durante la crisis de crédito en México y Chile. Este artículo muestra como cambiaron estimados lineales usando el método de momentos bajo diferentes supuestos durante el período de estudio.

STOCHASTIC DISCOUNT FACTOR FOR MEXICO AND CHILE, 2007-2012

The stochastic discount factor is a term used extensively in the financial literature to refer to adjustments for risk. This article explores how a linear stochastic discount factor has changed in the Mexican and Chilean economies during the period 2007-2012, which includes the period of the international economic credit crises 2008-2009.

The main uses for the stochastic factor are in asset pricing theory to do valuations and to assess market efficiency. Lucas (1978), Rubinstein (1976), Breeden (1979) and Cox, Ingersoll, et al. (1985) proposed and analyzed inter-temporal asset pricing models. Market efficiency has been analyzed using different approaches. The most frequent use is through asset pricing models. At this respect Valencia-Herrera (2012) uses the three and four linear factor model to analyze the returns of the Mexican sustainable index. Marquez de la Cruz (2006) uses the Consumption Based Asset Pricing Model (CCAPM) to analyze the permanent and non-permanent consumption in the Spanish Economy. Previously, Marquez de la Cruz (2005) estimated the inter-temporal rate of substitution for the Spanish Economy. Nieto y Rodriguez (2005) shows how to applied the CCAPM and the Fama and French (1996) three factor linear model to the Spanish and American Economies. Other analysis are, for example, Hansen and Jagannathan (1991), which estimated a lower bound on the volatility of the stochastic discount factor. Among the applications, it has been applied, for example, to measure the performance of fund managers, see (Farnworth, Ferson et al. 2002). They found that the method results in a small bias to lower returns.

Acuña and Pinto (2007) discuss volatility tests in the Chilean market using the Fama model for the period 2003-2007. Ahned, Haider et al. (2012) estimate the discount factor using consumption data in selected countries in Asia. Bogle, Feng et al. (2008) uses the stochastic discount factor to price derivative securities. Edge (2011) uses the stochastic discount factor to estimate the value of options. Burnside (2010) proposes a methodology to select normalizations when using the CCAPM to estimate stochastic discount factors. Gregoriou, Ionnidis et al. (2009) extends the Campbell and Shiller (1988) CCAPM model using the dividend price ratio to include transaction costs in a sample of panel data of advanced economies in the period 1984-2005. They found that heteroscedasticity changes depending on the transaction costs in the country.

The paper is divided in five sections. This one is an introduction. Section two introduces the moment conditions starting from a simple representative consumer-investor problem. Section three gives an overview of the Mexican and Chilean economies during the period of study. Section four includes the analysis and discussion of the empirical results. Then, the conclusions section follows.

1.- An analysis of equilibrium conditions

In the problem of a consumer, who can trade freely in assets i and who maximizes the expected value of a discounted time-separable utility.

$$Max \mathrm{E}_t[\sum_{j=0}^{\infty} \delta^j \mathrm{U}(\mathrm{C}_{t+j})]$$

where δ measures the personal time preference, C_{t+j} is the investor´s consumption in period $t+j$, and $U(C_{t+j})$ is the period utility of consumption at $t+j$, and wealth W_t at t satisfied the following relation

$$W_{t+1} = \sum_{i=1}^{I}(R_{i,t} - R_{f,t}) * w_{i,t} + R_{f,t})(W_t - C_t)$$

where $w_{i,t}$ is the proportion invested in risky asset i of the total wealth in period t, $R_{i,t}$ is the return of risky asset i in period t and $R_{f,t}$ is the return of the risk free asset in period t.

The optimal consumption and portfolio plan must satisfy that the marginal utility of consumption today is equal to the expected marginal utility benefit from investing one monetary unit in asset i at time t, selling it at time $t+1$ for $R_{i,t+1}$ and consuming the proceeds,

$$U'(C_t) = \delta E_t(R_{i,t+1} U'(C_{t+1}) \middle| \Psi_t) \qquad (1)$$

Dividing both sides in (1) by $U'(C_t)$, we get

$$1 = E_t(R_{i,t+1} M_{t+1} \mid \Psi_t) \qquad (2)$$

where the stochastic discount factor M_{t+1} is equal to the stochastic inter-temporal rate of substitution $\delta U'(C_{t+1}) / U'(C_t)$.

The expectation is conditional on the information available at time t in (2). By taking the unconditional expectations in (2) and rewriting the expression for period t, we obtain the unconditional version:

$$1 = E(R_{i,t} M_t) \qquad (3)$$

Rearranging, the expected asset returns can be estimated, noticing that $E(R_{i,t} M_t) = E(R_{i,t}) E(M_t) + Cov(R_{it}, M_t)$, so

$$E(R_{it}) = \frac{1 - Cov\left(R_{it}, M_t\right)}{E(M_t)} \tag{4}$$

If there is an unconditional zero beta asset, one whose unconditional covariance with the stochastic discount factor is zero, $E(R_{0,t}) = 1/E(M_t)$, the expression of the excess return Z_{it} on assset i over the zero-beta return is:

$$E(Z_{it}) = E(R_{it} - R_{0t}) = -E(R_{0t})Cov(R_{it}, M_t) \tag{5}$$

Note that if the returns of the n risky assets in the economy are the vector R_t, and $\overline{1}$ is a vector of ones, relationship (2) can be written as

$$\overline{1} = E\left(R_t M_t\right) \tag{6}$$

where R_t has an unconditional non-singular variance-covariance matrix Σ.

An implication of this model and other inter-temporal asset pricing ones is that

$$E(R_{t+1} \mid \Psi_t] - R_t^f = \frac{Cov(R_{t+1}, m_{t+1} \mid \Psi_t)}{E(m_{t+1} \mid \Psi_t)}$$

where the return on one period riskless bond is $R_t^f = 1/E(m_{t+1} \mid \mathrm{A}_t)$ and Ψ_t is a subset of the information set A_t and $R_t^f \in \Psi_t$.

For example, in the case of power utility, $U(C_t) = (C_t^{1-\gamma} - 1)/(1-\gamma)$, where γ is the coefficient of risk aversion. A limitation of the power utility is that the elasticity of inter-temporal substitution Φ is the reciprocal of the coefficient of relative risk aversion γ, which Hall(1988) argues that it is inappropriate because Φ concerns with the willingness to substitute consumption today with one of the future, whereas γ refers to the willingness of the investor to move consumption between possible future states of the world. Epstein and Zin (1991) and Weil (1989) break out the dichotomy. They propose a more general utility specification, which preserves the scale-invariance of the power utility, but breaks the equivalence between coefficient of relative risk aversion and elasticity of inter-temporal substitution.

If the information set is normal, any payoff satisfies

$$1 = E(m_{t+1}R_{t+1} \mid \Psi_t) = exp[E(log(m_{t+1}r_{t+1}) \mid \Psi_t) + \frac{1}{2}Var(log(m_{t+1}r_{t+1}) \mid \Psi_t)]$$

which can be written as:

$$E(log(r_{t+1})|\Psi_t) = E(log(m_{t+1})|\Psi_t) - \frac{1}{2}Var(log(m_{t+1}r_{t+1})|\Psi_t) \quad (7)$$

If the One Factor Capital Asset Pricing Model is satisfied,

$$E(R_{it}) = R_t^f + \beta_i\gamma = R_t^f + \beta_i(E(R_t^m) - E(R_t^f)) \quad (8)$$

where γ is a benchmark´s risk premium.

Assume that the stochastic discount factor m_t has the form $a + bR_t^m$,

Relation (2) can be written for R_t^f and R_t^m as

$$1 = E(m_t R_{it}) = E(m_t R_t^m) = E(m_t)R_t^f \quad (9)$$

From (9) and the expression for m,

$$b = \frac{1}{E\left(R_t^f\right)} - a\frac{1}{E\left(R_t^m\right)} \quad (10)$$

Replacing in $1 = E(m_t R_t^m)$ the expression for m_t and (10), noticing that $E\left(\left(R_t^m\right)^2\right) = \sigma^2\left(R_t^m\right) + E\left(R_t^m\right)^2$ and after solving for a, we get

$$a = \frac{1}{E\left(R_t^f\right)} + \frac{E\left(R_t^m\right)}{E\left(R_t^f\right)}\frac{\left(E\left(R_t^m\right) - E\left(R_t^f\right)\right)}{\sigma^2\left(R_t^m\right)} \quad (11)$$

Replacing (11) in (10),

$$b = \left(1 - \frac{E\left(R_t^m\right)}{E\left(R_t^f\right)}\right)\left(\frac{1}{\sigma^2\left(R_t^m\right)}\right) \quad (12)$$

The benchmark portfolio can be estimated maximizing the Sharpe ratio S_t,

$$S_t = \frac{w_t^T(R_t - R_t^f)}{w_t^T\Sigma w_t} \quad (13)$$

where $R_t^m = w_t^{*T}R_t$ and w_t^* are the optimum weights.

1.1 Estimation of Euler Equation of Consumption

In equilibrium, the conditional moment condition that the stochastic discount factor m_t must satisfy conditional on previous information Ψ_{t-1} is that the expected product of any return R_t with the discount factor must be equal to one,

$$E(m_t R_t \mid \Psi_{t-1}) = 1 \qquad \textbf{(14)}$$

In particular, deviations in the moment condition can be interpreted as return´s alpha for the investor, as in Chen and Knez (1996), or selection of an inappropriate discount factor. That is

$$\alpha = E(m_t R_t \mid \Psi_{t-1}) - 1 \qquad (15)$$

The Euler equation of consumption (14) shows the expected rate of return on the assets as well as relative expected consumption stream which is negatively related to the risk aversion parameter.

$$1 = E\left(R_t \delta_t \frac{c_{t+1}^{\gamma-1}}{c_t^{\gamma-1}} \right) = E\left(R_t m_t\right) \qquad (16)$$

This is what shows if the consumers prefer to trade-off their consumption in the present with more of it in the future. In order to estimate preference parameters of the Euler equation, the constant relative risk aversion coefficient (CRRA) γ and discount factor δ, the GMM technique is used. The necessary condition for GMM method to estimate the structural parameters is that the moment must hold.
To get the moment condition from equation (1) it is necessary to rearrange this equation as:

$$E\left(R_t \delta_t \frac{c_{t+1}^{\gamma-1}}{c_t^{\gamma-1}} \right) - 1 = E\left(R_t m_t\right) - 1 = 0 \qquad (17)$$

According to Hansen and Singleton (1982) the discrete-time models of the optimization behavior of economic agents often lead to a first-order conditions of the form:

$$E_t\left(h(x_t, b_o)\right) = 0 \qquad (18)$$

where x_t is a vector of variable observed by agents at time t and b_o is a p dimensional parameter vector to be estimated. Therefore:

$$E\left(h_t\left(x_t,b_o\right)\right)=E\left(R_t\delta_t\frac{c_{t+1}^{\gamma-1}}{c_t^{\gamma-1}}\right)-1=E\left(R_tm_t\right)-1=0 \qquad (19)$$

Let us construct an objective function that depends only on the available information of the agents and unknown parameters b. Let $g_0(b)=E[f(x_t;z_t;b_o)]$ according to Singleton (1982), if the model in (16) is true then the method of moment estimator of the function g is:

$$\mathrm{g}_T\left(b\right)=\frac{1}{T}\sum_{t=1}^{T}f\left(x_t,z_t,b\right) \qquad (20)$$

The value of $gT(b)$ at $b=b0$ should be close to zero for large values of T. In this paper, we follow Hansen and Singleton (1982) and choose b to minimize the function J_t

$$J_T(b)=g_T'(b)W_Tg_T(b) \qquad (21)$$

where W_T is a symmetric, positive definite weighting matrix W_T can be estimated minimizing

$$W_T=\frac{1}{T}\sum_{t=1}^{T}(f(x_t;z_t;b)f(x_t;z_t;b))=0 \qquad (22)$$

The choice of weighting matrix W_T is such that which makes g_T close to zero, taking into account possible heteroscedasticity and autocorrelation (HAC) behavior.

There are two advantages of estimating non-linear Euler equation under GMM as given in Hansen and Singleton (1982):

(a) Unlike the maximum likelihood (ML) estimator, the GMM estimator does not require the specification of the joint distribution of the observed variables.

(b) The instrument vector does not need to be economically exogenous. The only requirement is that this vector be predetermined in the period when the agent forms his expectations. Both past and present values of the variables in the model can be used as instruments. Model estimator is consistent even when the instruments are not exogenous or when the disturbances are serially correlated.

To compute W_T a consistent estimator of b_o is needed. This can be obtained by initially using $W_t=I_{r\times r}$ (identity matrix) and suboptimal choice of b in minimizing $J_t(b)$ (18) and we get the values of b_T. By using this value of b in (19) we get W_T. Again by using the new values of W_T, b_T can be obtained by minimizing equation (18). We repeat this process until the estimates converge. According to Puzzi (2003) this iterative GMM process is more efficient in small sample than a simple standard two-step procedure given by Hansen and Singleton (1982).

3.- The Mexican and Chilean economies

3.1 The Mexican economy

In the period of study five sup-periods can be identified: a slowdown of the economy, during 2007 and 2008, the crises in Mexico, at the end of 2008 and beginning of 2009, the recovering period, 2009, 2010 and 2011 and a slowdown of the economy, at the end of 2012. During 2007, the economy slowed down because the credit crises in the United States weakened their economy, Mexican exports moderated their growth and commodity prices increased: oil, food and metallic supplies suffered inflation. In august of 2008, the international banking market crises aggravated. With the bankruptcy of Lehman Brothers in September, uncertainty in the international market grew. The international markets lacked of liquidity. The crises expanded to other financial markets, including the Mexican. By the second quarter of 2008, the crises effects began to subside. The measures that Mexican and international authorities had implemented started to have effects. Progressively liquidity to markets increased and the uncertainty diminished and growth return to the Mexican economy. During 2012, the uncertainty derived from the European Crises affected the American Economy. Mexican exports slowed down and the manufacturing activity in some regions of Mexico contracted. There were signs of a possible deterioration of the economic activity prospects in México.

3.2 The Chilean economy

The Chilean government conducts a rule-based contra cyclical fiscal policy, accumulating surpluses in sovereign wealth funds during periods of high copper prices and economic growth, and allowing deficit spending only during periods of low copper prices and growth.

Chile had a mild economic crisis as a consequence of the world wide credit crises. Chile benefited from a governmental rule-based countercyclical fiscal policy, accumulating surpluses in sovereign wealth funds during periods of high copper prices and economic growth, and having deficit spending during downturn periods. The economics went from a recuperation period in 2006 and 2007 to slowdown period of 2008 and suffer the world wide crises consequences in 2009. However by 2010, the Chilean economy has fully recovered. During the period of 2010 to 2012, it grows 6%, each year. In 2012, in spite of the European crises, the Chilean economy sustained its growth.

Inflation reduced gradually during 2006 to 2008, from being 13% in 2006 to 5% and 1% in 2007 and 2008. In 2009 and 2010, as a consequence of the international economic crises and the contra cyclical expansionary measures, inflation rebounds to 4% and 7%, respectively. For 2011 and 2012, prices stabilized, inflation only grew 3% and 2%, respectively.

4.- Discussion and Analysis

Mexico and Chile are two of the more open economies in Latin America, which have a high degree of integration with the rest of the world. The world credit crises affect affected directly the United States, main commercial partner of Mexico, and eventually many European countries. Chile has strong commercial links with both economies. Many companies in both countries depend on United States and European funding. Many American and European international enterprises operate in both countries. We expected that the stochastic discount factor changed in these economies during the crises period due to contagion effects.

In this study we analyze the performance of the Mexican Stock Market and the Chilean Stock Market. In each market a a market index is selected as benchmark. The index used in the Mexican Market was the Total Return Index "Índice de Rendimiento Total (IRT)" and for the Chilean Market, the Santiago Stock Exchange Index "Índice de la Bolsa de Santiago "IPSA", both indexes adjusted inclusive for cash dividends. The return and standard deviation of these indexes each year in the period of study is showed in Table 1.

Table 1 shows the mean and standard deviation of the returns during the period of study, and each of the years. Notice that the expected returns of IPSA and M-Chile and the ones of IRT and M-Mexico are very similar each year, however IPSA and IRT have more volatility than M-Chile and M-Mexico, measured by its standard deviation. Notice two that 2008 had negative returns measured by IPSA and IRT. The same happen in 2011, when the prospects of the Mexican and Chilean economies weakened. The recovery was stronger during 2009 and 2010. The growth in 2012 was small, compared with those of 2009 and 2010. Volatility increased in 2008, decreased in the following two years, and it increased again in 2011, and has a slowdown in 2012, in both the IPC and the IRT.

Two equations were estimated using method of moments. If equation (9) is estimated for each return and the return for the risk free rate is subtracted for each of the returns, the following moment condition must be satisfied:

$$E\left(m_t\left(R_{it}-R_t^f\right)\right)=E\left(m_t R_{i,t}^e\right)=0 \qquad (23)$$

where $R_{i,t}^e$ is the excess return of asset i. Using the fact that m_t can be written as $a+bR_{i,t}^e$, and standardizing the coefficient a as 1, as in Kosi (2006), we get the one parameter model

$$E\left(m_t R_{i,t}^e\right)=E\left(\left(1+bR_{m,t}^e\right)R_{i,t}^e\right)=E\left(R_{i,t}^e\right)+bE\left(R_{i,t}^e R_{m,t}^e\right)=0 \qquad (24)$$

The two parameter model comes from equation (15), where α can be different from zero, in this case the moment condition becomes

$$E\left(m_t R_{i,t}^e\right) - a_0 = E\left(\left(1 + bR_{m,t}^e\right)R_{i,t}^e - a_0\right) = E\left(R_{i,t}^e\right) - a_0 + bE\left(R_{i,t}^e R_{m,t}^e\right) = 0 \quad (25)$$

Table 2 shows the coefficients using two-step estimation each of the years of study for IPSA and IRT as market indexes for Mexico and Chile. Notice that 2008 and 2010 have a negative coefficient E(R*RME) for Mexico given that the expected returns were negative. The slope coefficient for Chile was small for those years compared with other years, in which the expected index return became negative. Also notice that the constant term, which measures a possible alpha return, was statistically significant for Mexico in 2011, and it was statistically significant for Chile in 2007, 2008, 2010, and 2011, but in 2010 it was positive. A negative coefficient signals a positive alpha return, which it can indicate a possible inefficient market or an inadequate model or index. The results suggest that a linear model with IRT can be more appropriate for Mexico than a linear one with IPSA for Chile.

Table 3 shows that results if different specification test for the two step model. Hansen overidentification test cannot reject the hypothesis that there is not overidentification except for 2007 and 2011 for the Mexican Model, and except for 2008, 2009 and 2011 for the Chilean Model. The Kleiderbergen-Paap rk LM tests for weak under-identification rejects the hypothesis of weak identification, except for Chile in 2010. The Kleiderbergen-Paap rk Wald F Test of weak underidentification signals that there are not weak identification problems except for the years 2009 and 2010 in the Mexican economy, 2008, 2009 and 2011 for the Chilean Economy. This is a test for which there are not critical value tables under heteroscedasticity and autocorrelation. For this reason, this is only and indicator.

The results of the iterated method of moments in Table 4 reaffirm the results observed in Table 2. There are statistically significant negative slope coefficients in the Mexican economy for 2008 and 2010, when the IRT became negative. In year 2010 there is a statistically significant negative alpha, and in year 2011 there is a statistically significant positive alpha. The constant coefficient is positive and negative, respectively. In Chile, during 2008 and 2009, the slope coefficients became smaller and there is a statistically significant positive alpha during the years 2007, 2008 and 2011 in Chile. Iterated method of moments did not converge for 2012, therefore results are not warranted.

Table 5 shows the results of two step estimation considering only a slope coefficient in the model, that is assuming that the moment condition is satisfied in the model. For Mexico, al coefficients are statistically significant except for 2010. The coefficient for 2008 becomes negative, given that the expected IRT return was negative. The coefficients for Chile all are statistically significant at 10% significant level, except for 2012. However, in 2008 and 2010, the results the coefficients are not statistically significantly different from cero at 5% of confidence interval. Iterated GMM reaffirms the previous results. For Mexico, the slope coefficient is negative and statistically significant in 2007 and 2008. It is positive from 2009 to 2011. For

Chile, all coefficients are negative and statistically significant. However, they are smaller in 2010 and 2012.

The sensitivity of the discount factor to the market factor premium changed differently in Mexico and in Chile during the crises periods. The sensitivity of the stochastic discount factor to the market factor premium became positive in México from 2009 to 2011, which is contra-intuitive. The sensitivity returned to negative in 2012. Instead, in Chile, during the period after the crises the sensitivity increased substantially, for example in 2010 it became -256.2878 and in 2012 became -299.3197, see table 5. The discussion centered on the sensibility of the discount factor premium centered on the iterated method of moments results because they have better small sample properties than the two step ones.

Hansen over-identification tests for the two step method of moments estimation with one parameter shows that the hypothesis of over-identification must be rejected for all years except 2011 for Mexico and for all years except 2008 for Chile. This suggest that the model is correctly specified, see table 6.

Tables 7 and 8 respectively show the stochastic discount factor in the two and one parameter models. Notice that the results in the two step method of moments estimation in both models are similar for each country. In year 2009, 2010, and 2012 the volatility of the stochastic discounted factor increased for Mexico. In year 2007, 2009 and 2011, volatility of the factor increased for Chile. However results for the volatility of the discounted factor changed in the iterated method of moments changed, but the expected discount factor is similar to the one in the two step estimation. Further analysis is warrant for the understanding of the differences.

The expected stochastic discount factor became smaller in Mexico in 2009 (0.982) and in Chile in 2011 (0.981) supporting the hypothesis that the Mexican crises was directly related with the American credit crises and the Chilean crises is related the European crises in 2011, see table 8.

5.- Conclusions and recommendations

The stochastic discount factor changed during the previous crises credit period in the Mexican and Chilean economies. The changes were deeper in the Mexican economy than in the Chilean economy. Special care warrants the interpretation of results because misspecification of the stochastic discount factor using the method of moments can easily result in unrealistic estimates. Test of overidentification, underidentification and weak underidentification can help better to interpret results.

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Table 1 Mean and Standard Deviation of the Daily Market Returns in Mexico and Chile

Country Index	Chile: IPSA		México: IPC	
Year	Mean	Std.Dev.	Mean	Std.
2007-12	1.00046	0.012046	1.000591	0.014539
2007	1.000581	0.012185	1.000618	0.013519
2008	0.999169	0.018479	0.999248	0.022944
2009	1.001694	0.010248	1.001659	0.017057
2010	1.001304	0.007358	1.000818	0.009068
2011	0.999442	0.013888	0.99999	0.012328
2012	1.000136	0.005965	1.000739	0.007107

Daily returns. M-Mexico and M-Chile refers to an index that maximizes the Sharpe Ratio in México and Chile, respectively, based on historical previous year returns with at least 50 quotes.

Table 2 Two step method of moments with market index IRT and IPSA for the Mexico and Chile Stock Exchanges, respectively

México	E(R* RME)			Constant		
	Coef.	z		Coef.	z	
2007	-0.668991	-0.08		0.0013316	1.13	
2008	-4.09106	-2.31	**	0.0009453	1.12	
2009	8.521878	0.7		0.0005383	0.19	
2010	-14.8147	-0.85		0.0016701	1.53	
2011	8.465221	4.49	***	-0.0008323	-2.5	**
2012	36.99887	2.22	**	-0.0001165	-0.2	

Chile	E(R* RME)			Constant		
	Coef.	z		Coef.	z	
2007	20.94971	9.18	***	-0.0015801	-5.88	***
2008	3.50058	3	***	-0.0011218	-3.25	***
2009	21.13738	3.3	***	-0.0000132	-0.03	
2010	11.22074	0.79		0.0012659	2.71	***
2011	18.63377	5.96	***	-0.002731	-6.35	***
2012	-24.35621	-1.06		0.0007628	1.35	

Table 3 Tests for the two step method of moments with market index IRT and IPSA for the Mexico and Chile Stock exchanges, respectively.

IRT 2step bw(optimum)						
México						
Year	2007	2008	2009	2010	2011	2012
kernel=Bartlett; bandwidth	62	61	62	63	63	63
Underidentification test (Kleibergen-Paap rk LM statistic):	36.43	35.454	17.956	24.151	56.153	34.777
Chi-sq(3) P-val	0	0	0.0004	0	0	0
Weak identification test (Kleibergen-Paap rk Wald F statistic):	15.912	45.659	**7.318**	**8.669**	127.372	18.897
Hansen J statistic (overidentification test of all instruments):	7.254	3.954	4.54	2.642	23.073	5.716
Chi-sq(2) P-val	0.0266	0.1385	0.1033	0.2668	**0**	0.0574
Chile						
Year	2007	2008	2009	2010	2011	2012
kernel=Bartlett; bandwidth	64	62	62	66	64	64
Underidentification test (Kleibergen-Paap rk LM statistic):	116.424	63.45	42.492	4.984	58.394	23.496
Chi-sq(3) P-val	0	0	0	0.173	0	0
Weak identification test (Kleibergen-Paap rk Wald F statistic):	148.081	107.249	27.767	24.989	45.336	9.778
Hansen J statistic (overidentification test of all instruments):	1.748	35.956	6.344	1.707	10.484	2.851
Chi-sq(2) P-val	0.4173	**0**	0.0419	0.4258	**0.0053**	0.2404

Stock-Yogo weak ID test critical values:	
5% maximal IV relative bias	13.91
10% maximal IV relative bias	9.08
20% maximal IV relative bias	6.46
30% maximal IV relative bias	5.39
10% maximal IV size	22.3
15% maximal IV size	12.83
20% maximal IV size	9.54
25% maximal IV size	7.8

Source: Stock-Yogo (2005). Reproduced by permission.

NB: Critical values are for Cragg-Donald F statistic and i.i.d. errors

Table 4 Iterated method of moments with market index IRT and IPSA for the Mexico and Chile Stock Exchanges, respectively

México	E(R* RME)			Constant		
	Coef.	Z		Coef.	z	
2007	-2.875062	-0.35		0.0016136	1.37	
2008	-4.991535	-2.98	***	0.0012179	1.48	
2009	-3.989036	-0.46		0.0029774	1.51	
2010	-20.88189	-1.33		0.0020475	2.07	**
2011	8.911533	4.72	***	-0.0008195	-2.47	**
2012	41.53322	2.49	**	-0.0002644	-0.45	
Chile	E(R* RME)			Constant		
	Coef.	Z		Coef.	z	
2007	20.98917	9.2	***	-0.0015838	-5.89	***
2008	4.426769	3.75	***	-0.0011908	-3.52	***
2009	20.40781	3.19	***	0.0000166	0.04	
2010	66.3152	5.51	***	-0.0006773	-1.65	
2011	15.73101	6.39	***	-0.0024712	-6.53	***
2012	-22.58119	-0.99		0.000721	1.29	

Table 5.- One parameter estimate in two step estimation with dependent variable IRT and IPSA, for Chile and México, respectively.

México				Chile			
2 step				2 step			
	E(R* RME) Coef	z			E(R*RME) Coef.	z	
2007	6.973593	3.02	***	2007	14.60526	9.41	***
2008	-3.349655	-2.9	***	2008	1.531666	1.67	*
2009	9.668524	2.73	***	2009	21.12897	6.23	***
2010	1.333474	0.27		2010	22.61786	1.8	*
2011	6.477939	4.83	***	2011	13.35944	5.63	***
2012	29.66323	3.51	***	2012	-7.260106	-0.56	
Iterated GMM				Iterated GMM			
2007	-30.7588	-5.5	***	2007	-31.23999	-11.05	***
2008	-11.87998	-5.83	***	2008	-5.898807	-6.33	***
2009	12.69343	2.16	**	2009	-51.71992	-6.08	***
2010	15.2248	5.96	***	2010	-256.2178	-5.13	***
2011	15.2248	5.96	***	2011	-27.41669	-9.54	***
2012	-9.025548	-0.49		2012	-299.3197	-5.21	***

Table 6.- Overidentification test in 2 step estimation with dependent variable IRT and IPSA, for Chile and México, respectively, and one parameter estimate.

México			Chile		
2 step	Hansen J*	Chi-sq(2) P-val	2 step	Hansen J*	Chi-sq(2) P-val
2007	6.903	0.0317	2007	0.523	0.7699
2008	3.792	0.1502	2008	36.839	0
2009	4.486	0.1061	2009	6.351	0.0418
2010	3.032	0.2196	2010	2.541	0.2807
2011	21.217	0	2011	8.621	0.0134
2012	5.478	0.0646	2012	3.394	0.1832

*Ho There is not overidentification with all instruments

Table 7 Stochastic Discount Factor based on a one factor variable, 2 step method of moments and iterated method of moments (two coefficient model).

México	IPC		IPSA	
Variable	Mean	Std.Dev.	Mean	Std.Dev.
2007	1.000	0.009	0.991	0.255
2008	0.996	0.094	1.004	0.065
2009	0.988	0.145	0.965	0.217
2010	1.008	0.134	0.986	0.083
2011	1.003	0.104	1.013	0.259
2012	0.984	0.263	1.000	0.145

Iterated

México	IPC		IPSA	
Variable	Mean	Std.Dev	Mean	Std.Dev.
2007	1.001	0.039	0.991	0.256
2008	0.995	0.115	1.005	0.082
2009	1.006	0.068	0.967	0.209
2010	1.013	0.189	0.916	0.488
2011	1.002	0.110	1.011	0.218
2012	0.975	0.295	1.000	0.135

Mean = 1-b*(Rm-Rf); Std. Dev. =abs(b)*Std. Dev.(Rm)

Table 8 Stochastic Discount Factor (one parameter model)

2 STEP

México	IPC Mexico		IPSA Chile	
Variable	Mean	Std.Dev.	Mean	Std.Dev.
2007	0.998	0.094	0.994	0.178
2008	0.996	0.077	1.002	0.028
2009	0.986	0.165	0.965	0.217
2010	0.999	0.012	0.971	0.166
2011	1.001	0.080	1.009	0.186
2012	0.982	0.211	1.000	0.043

Iterated

México	IPC		IPSA	
Variable	Mean	Std.Dev.	Mean	Std.Dev.
2007	1.010	0.416	1.013	0.381
2008	0.987	0.273	0.994	0.109
2009	0.982	0.217	1.085	0.530
2010	0.991	0.138	1.324	1.885
2011	1.003	0.188	0.981	0.381
2012	1.005	0.064	0.999	1.785

Mean = 1-b*(Rm-Rf); Std. Dev. =abs(b)*Std. Dev.(Rm)